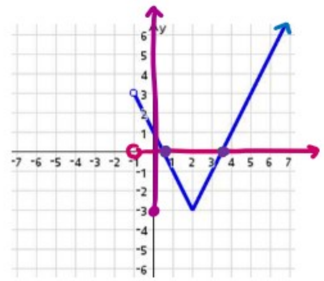
Characteristics of Functions



Domain:
(x values - left to right)
(-1, ∞)

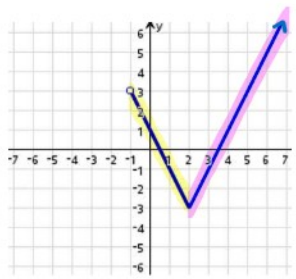
Range:
(y values - bottom to top)
[-3, ∞)

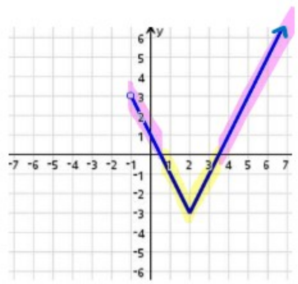
Zero(s):
(x-intercept)
(.5, 0) (3.5, 0)

End Behavior:
(describes what is happening on left and right sides)

As $x \to -\infty, f(x) \to$ DNE

As $x \to +\infty, f(x) \to$ ∞





Increasing Interval(s):
(graph goes up from left to right)
(2, ∞)

Decreasing Interval(s):
(graph goes down from left to right)
(-1, 2)

Positive Interval(s):
(graph is above the x-axis)
(-1, .5) ∪ (3.5, ∞)

Negative Interval(s):
(graph is below the x-axis)
(.5, 3.5)

Transformation Equation

$$g(x) = a \cdot f\left(\frac{1}{b}(x-h)\right) + k$$

bx + h	x	$f(x)$	ay + k
new x	parent function values		new y

Vertical Transformations

(outsiders: a and k)

a: vertical dilation and reflection

$a < 0$ *(a is negative)*	reflect over x-axis

$\lvert a \rvert = 1$	no change
$0 < \lvert a \rvert < 1$	vertical shrink
$\lvert a \rvert > 1$	vertical stretch

k: vertical translation

$k = 0$	no change
$k < 0$ *(k is negative)*	down
$k > 0$ *(k is positive)*	up

Horizontal Transformations

(insiders: b and h)

b: horizontal dilation and reflection, liar - reciprocal

$b < 0$ *(b is negative)*	reflect over y-axis

$\lvert b \rvert = 1$	no change
$0 < \lvert b \rvert < 1$	horizontal shrink
$\lvert b \rvert > 1$	horizontal stretch

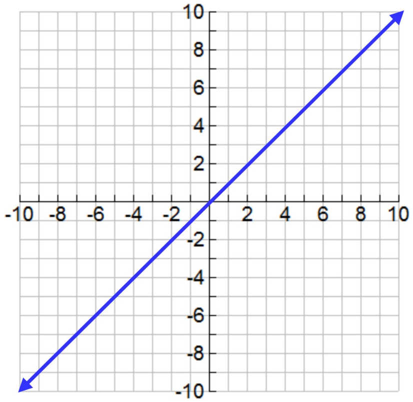
h: horizontal translation, liar - opposite

$h = 0$	no change
$h < 0$ *(h is negative)*	left
$h > 0$ *(h is positive)*	right

Parent Function Name: **Linear**

Equation: $f(x) = x$

Graph:



x	$f(x)$
-2	-2
-1	-1
0	0
1	1
2	2

Transformation Equation:

$y - y_1 = m(x - x_1)$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Needed to Write Equation:

Point (x_1, y_1)
Slope m

Additional Notes:

Slope

$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Horizontal Line
$y = \#$
$m = 0$

Vertical Line
$x = \#$
$m =$ undefined

Slope-Intercept Form
$y = mx + b$

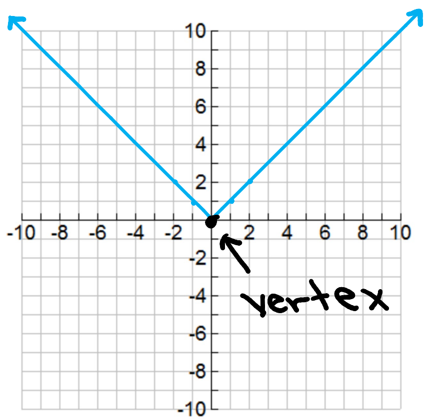
Point-Slope Form
$y - y_1 = m(x - x_1)$

Standard Form
$Ax + By = C$

Parent Function Name: **Absolute Value**

Equation: $f(x) = |x|$

Graph:



x	$f(x)$
-2	2
-1	1
0	0
1	1
2	2

Transformation Equation:

$$y = a\left|\frac{1}{b}(x-h)\right| + k$$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

$a > 0$	$a < 0$
$[k, \infty)$	$(-\infty, k]$

Needed to Write Equation:

Let $b = 1$

vertex (h, k)

point (x, y)

solve for a

Additional Notes:

Solve Absolute Value Equations / Inequalities

1. Isolate
2. Separate
   - drop
   - drop and flip
3. Solve

greatOR

less thAND

Equation	Inequality
4. Check	4. Combine AND Graph OR

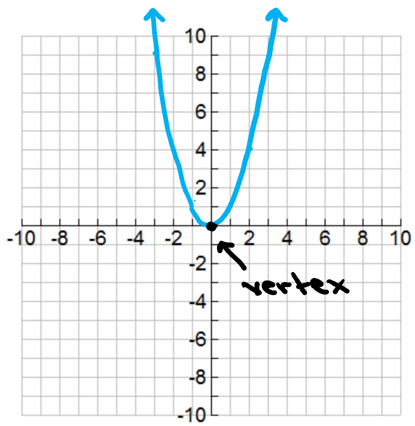
Word Problems

$|x - \text{actual}| \leq \text{tolerance}$

Parent Function Name: **Quadratic**

Equation: $f(x) = x^2$

Graph:



x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

Transformation Equation:

$$y = a\left(\frac{1}{b}(x-h)\right)^2 + k$$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

$a > 0$	$a < 0$
$[k, \infty)$	$(-\infty, k]$

Needed to Write Equation:

vertex form $y = a(x-h)^2 + k$ vertex (h,k) and point (x,y) or axis of symmetry $x=h$ and two points

intercept form $y = a(x-p)(x-q)$ x-int $(p,0)$ and $(q,0)$ and point (x,y)

standard form $y = ax^2 + bx + c$ quadratic regression

Additional Notes:

vertex/intercept → standard
expand (multiply)

standard → vertex
complete the square

standard → intercept
factor

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Imaginary Numbers

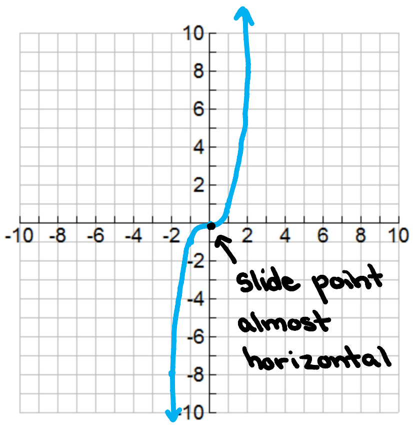
$i = \sqrt{-1}$

$i^2 = -1$

Parent Function Name: **Cubic**

Equation: $f(x) = x^3$

Graph:



x	$f(x)$
-2	-8
-1	-1
0	0
1	1
2	8

Needed to Write Equation:

Let b = 1

slide point (h, k)

point (x, y)

solve for a

Additional Notes:

Transformation Equation:

$$y = a\left(\frac{1}{b}(x-h)\right)^3 + k$$

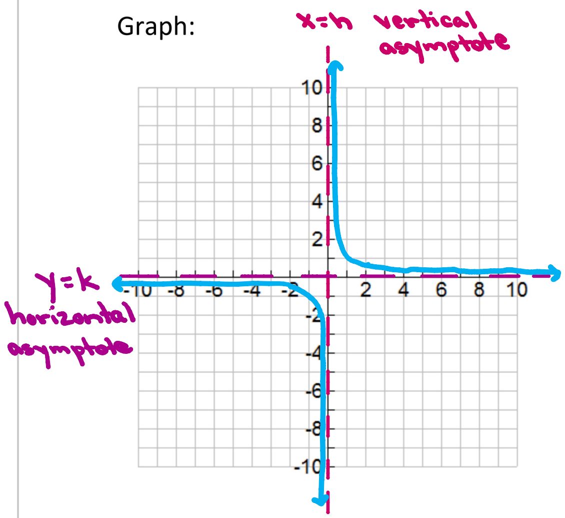
Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Parent Function Name: **Rational**

Equation: $f(x) = \frac{1}{x}$

Graph:



x	$f(x)$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	DNE
$\frac{1}{2}$	2
1	1
2	$\frac{1}{2}$

Transformation Equation:

$$y = \frac{a}{\frac{1}{b}(x-h)} + k$$

Domain: $(-\infty, \overset{h}{0}) \cup (\overset{h}{0}, \infty)$

Range: $(-\infty, \overset{k}{0}) \cup (\overset{k}{0}, \infty)$

Needed to Write Equation:
Let $b = 1$
horizontal asymptote $y = k$
vertical asymptote $x = h$
point (x, y)
solve for a

Additional Notes:

General Rational Functions

Horizontal Asymptote

BOBO - bigger on bottom, $y = 0$

BOTN - bigger on top, none

EATS DC - exponents are the same, divide (leading) coefficients

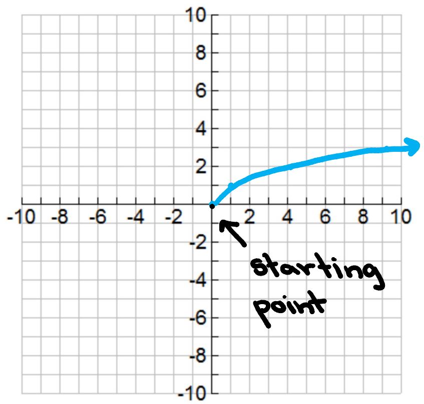
Vertical Asymptote(s)

set denominator $= 0$, solve for x

Parent Function Name: **Square Root**

Equation: $f(x) = \sqrt{x}$

Graph:



x	$f(x)$
-4	DNE
-1	DNE
0	0
1	1
4	2

Needed to Write Equation:

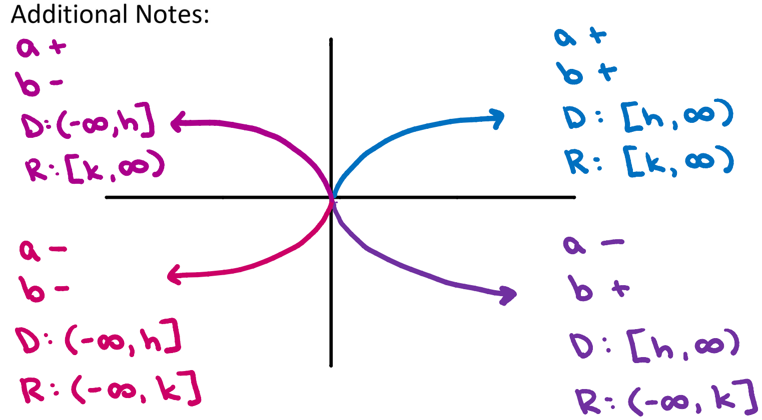
Let b=1 or a=1

starting point (h,k)

point (x,y)

Solve for a or b

Additional Notes:



Transformation Equation:

$$y = a\sqrt{\frac{1}{b}(x-h)} + k$$

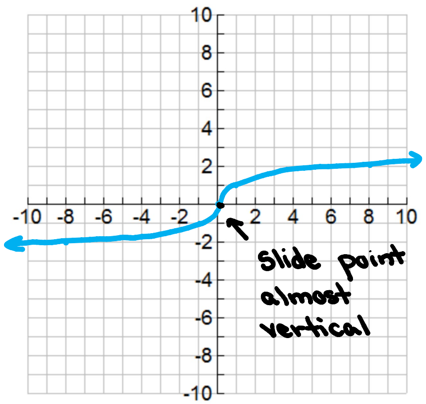
Domain: $[0, \infty)$

Range: $[0, \infty)$

Parent Function Name: **Cube Root**

Equation: $f(x) = \sqrt[3]{x}$

Graph:



x	$f(x)$
-8	-2
-1	-1
0	0
1	1
8	2

Transformation Equation:

$$y = a\sqrt[3]{\frac{1}{b}(x-h)} + k$$

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Needed to Write Equation:

Let b = 1

slide point (h, k)

point (x, y)

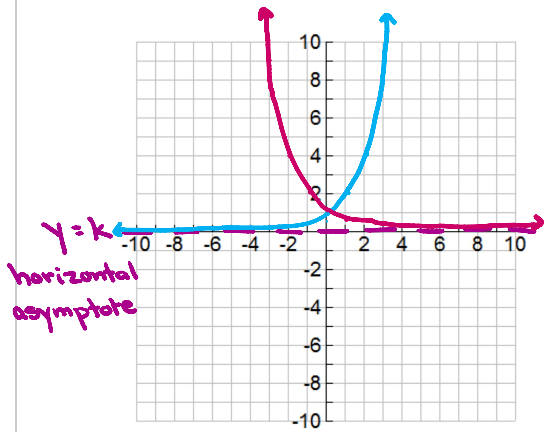
solve for a

Additional Notes:

Parent Function Name: **Exponential**

Equation: $f(x) = B^x$

Graph:



x	$f(x)$	$y = 2^x$ growth	$y = (\frac{1}{2})^x$ decay
-1	$\frac{1}{B}$	$\frac{1}{2}$	2
0	1	1	1
1	B	2	$\frac{1}{2}$

Transformation Equation:

$$y = a(B)^{\frac{1}{b}(x-h)} + k$$

Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

$a > 0$	$a < 0$
(k, ∞)	$(-\infty, k)$

Needed to Write Equation:

Additional Notes:

Growth $B > 1$

Decay $0 < B < 1$

B cannot be negative, 0, or 1

Euler's Number

$e \approx 2.71828$

Growth / Decay

$y = a(1 \pm r)^x$

Compound Interest

$A = P(1 + \frac{r}{n})^{nt}$

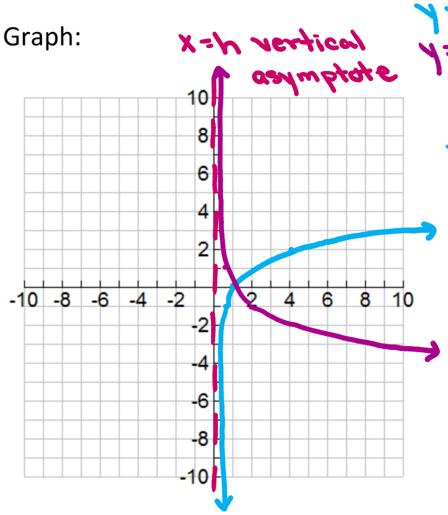
Continuously Compounded Interest

$A = Pe^{rt}$

Parent Function Name: **Logarithmic**

Equation: $f(x) = \log_B x$

Graph:



$y = \log_2 x$ increasing

$y = \log_{\frac{1}{2}} x$ decreasing

		x	$f(x)$
$\frac{1}{2}$	2	$\frac{1}{B}$	-1
1	1	1	0
2	$\frac{1}{2}$	B	1

Transformation Equation:

$$y = a\, log_B \left(\frac{1}{b}(x-h)\right) + k$$

Domain: $(0, \infty)$

$b > 0$ (h, ∞)

$b < 0$ $(-\infty, h)$

Range: $(-\infty, \infty)$

Needed to Write Equation:

Additional Notes:

$\log_B m^n = n \log_B m$

$\log_B mn = \log_B m + \log_B n$

$\log_B \frac{m}{n} = \log_B m - \log_B n$

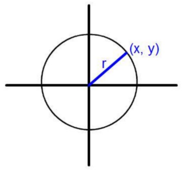
$\log_B a = \frac{\log a}{\log B} = \frac{\ln a}{\ln B} = \frac{\log_c a}{\log_c B}$

$\log_B x = y \iff B^y = x$

Circle

Center (h, k) Radius $\sqrt{r^2}$

$$(x-h)^2 + (y-k)^2 = r^2$$



Ellipse

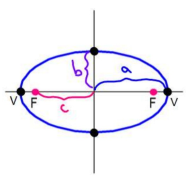
Center (h, k) $a > b$

a = major radius
b = minor radius

c = focal radius
$c = \sqrt{a^2 - b^2}$

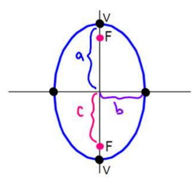
Horizontal

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



Vertical

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$



2 Vertices – a units from the center on the major axis
2 Foci – c units from the center on the major axis

Hyperbola

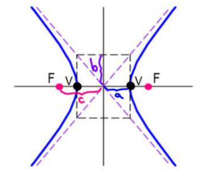
Center (h, k)

a = transverse radius
b = conjugate radius

c = focal radius
$c = \sqrt{a^2 + b^2}$

Horizontal

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

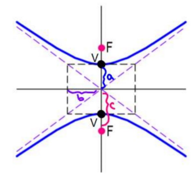


Asymptotes

$$y - k = \pm\frac{b}{a}(x-h)$$

Vertical

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



Asymptotes

$$y - k = \pm\frac{a}{b}(x-h)$$

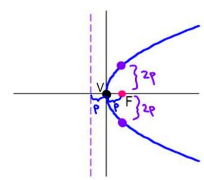
2 Vertices – a units from the center on the transverse axis
2 Foci – c units from the center on the transverse axis

Parabola

Vertex (h, k)

Horizontal

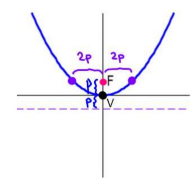
$$(y-k)^2 = 4p(x-h)$$



$p_x < 0$ opens left
$p_x > 0$ opens right

Vertical

$$(x-h)^2 = 4p(y-k)$$



$p_y < 0$ opens down
$p_y > 0$ opens up

Focus – a point p units from the vertex inside the parabola
Directrix – a line p units from the vertex outside the parabola